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Real Estate Valuation Models: Lender and Equity  
Investor Criteria

Roger E. Cannaday  
Peter F. Colwell

THE UNIVERSITY OF ILLINOIS  
COLLEGE OF COMMERCE AND BUSINESS ADMINISTRATION  
BUREAU OF ECONOMIC AND BUSINESS RESEARCH  
URBANA-CHAMPAIGN

College of Commerce and Business Administration  
Bureau of Economic and Business Research  
University of Illinois, Urbana-Champaign



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Lender and Equity Investor Criteria

Roger E. Cannaday, Assistant Professor  
Department of Finance

Peter F. Colwell, Associate Professor  
Department of Finance





## Abstract

The use of valuation models that focus on lender criteria has been growing in the appraisal field. In the rush to build lender criteria into real estate valuation models, equity investor criteria, expectations, and requirements occasionally have been ignored.

This paper integrates lender and equity investor criteria with traditional discounted cash flow models. The specific criteria considered are the loan-to-value ratio and the debt coverage ratio for lenders, and the equity dividend rate for equity investors. These three criteria are assumed to be binding constraints on value.

Graphical analysis provides a framework within which major real estate valuation models (i.e., Ellwood, McLaughlin, Gettel, Lusht-Zerbst, and Steele) are compared. A new valuation model (i.e., the Cannaday-Colwell model) is developed which utilizes the equity dividend rate.

The three definitional models (i.e., McLaughlin, Gettel, and Steele) are found to be correct only by mere coincidence. Each of these models simultaneously considers two of the three key criteria, completely eliminating the possibility of consideration of anything else; i.e., the models become tautological.

It is shown that the discounted cash flow based models (i.e., Ellwood, Lusht-Zerbst, and Cannaday-Colwell) each tell one-third of the story. One of these models will be correct depending upon whether the binding constraint is the maximum loan-to-value ratio, the minimum debt coverage ratio, or the minimum equity dividend rate. The correct model is the one that yields the lowest value estimate of the three.



## REAL ESTATE VALUATION MODELS: LENDER AND EQUITY INVESTOR CRITERIA

The use of valuation models that focus on lender criteria has been growing in the appraisal field. These lender criteria are the primary underwriting standards used by lenders. The reason that their use has grown in valuation models is that inflation has caused many loan applications to be rejected on the basis that criteria developed in an era of relative price stability could not be met. Thus, real estate lenders, investors, and appraisers have become sensitized to viewing these criteria as binding constraints.

Unfortunately, in the rush to build lender criteria into real estate valuation models, equity investor criteria, expectations, and requirements occasionally have been ignored. The equity investor criterion that has a status similar to the lender's underwriting standards is an initial rate of return measure. Even casual empiricism will indicate the importance of such "hurdle" rates in identifying feasible projects for investors.<sup>1</sup> The importance of building-in investor expectations concerning future income and property value movements as well as required internal rates of return (equity yield rates) is well established.

This paper uses graphical analysis to integrate these lender and equity investor criteria with traditional discounted cash flow models. The specific criteria considered are the loan-to-value ratio and the debt coverage ratio for lenders and the equity dividend rate for equity investors.<sup>2</sup> The graphical analysis provides a framework within which major real estate valuation models are compared (i.e., models developed by Ellwood, McLaughlin, Gettel, Lusht-Zerbst, Steele, and Fisher-Lusht).<sup>3</sup> It also provides insight to develop a new valuation model (i.e., the

Cannaday-Colwell model), one that utilizes an equity investor criterion within a discounted cash flow model. Finally, it provides the means to select a valuation model consistent with lender and equity investor criteria as well as the traditional concerns of discounted cash flow.

### Alternative Real Estate Valuation Models

Two of the models presented here have been available for over 20 years while the others have only recently appeared in the literature. The Ellwood and McLaughlin models both were first proposed in 1959, the Gettel model appeared in 1978, the Lusht-Zerbst model in 1980, and the Steele and Fisher-Lusht models in 1981; the Cannaday-Colwell model makes its first appearance in this paper.

#### The Ellwood Model

The Ellwood model is a discounted cash flow model that has been simplified for computational ease. One of the more prominent features of the Ellwood model is the role played by the loan-to-value ratio as a capital (i.e., credit) rationing device. The conventional Ellwood model is expressed as follows:

$$(1) \quad V = \frac{\text{NOI}}{(1 - M)y + Mf - MP/s_n^y + \text{dep}/s_n^y}$$

where the terms for this and all subsequent equations are as defined in Table 1.<sup>4</sup> The loan to value ratio (M) in the Ellwood model is assumed to be the maximum loan-to-value ratio allowed by lenders (i.e., one of the two lender criteria that are the focus of this paper). Implicitly, it is assumed that the mortgage constant (f) is exogenous to the valuation of any project and that the equity yield rate (y) and expectations

TABLE 1

Definition of Symbols

V = estimated value of total property

NOI = net operating income

y = equity yield rate

n = holding period

M = mortgage to value ratio stated as a proportion

MV = amount of mortgage

i = interest rate on mortgage

m = term of mortgage

$f = \frac{i(1+i)^m}{(1+i)^m - 1}$  = mortgage constant

$P = \frac{(1+i)^n - 1}{(1+i)^m - 1}$  = proportion of the mortgage paid off

$s_n^y = [(1+y)^n - 1]/y$  = future worth of 1 per period factor; subscript indicates number of periods and superscript indicates the discount rate

dep = proportion by which the property value is expected to depreciate during the holding period

$EDR = \frac{NOI - MVf}{(1-M)V}$  = equity dividend rate

DCR = NOI/MVf = debt coverage ratio

of net operating income (NOI), holding period (n), and appreciation (-dep) are those of the marginal equity investor.<sup>5</sup>

The Ellwood model can be derived from the statement that the present value of the property equals the present value of the mortgage plus the present value of the return to equity; hence it is often characterized as a mortgage-equity model. A recent survey by Lusht reveals that, "Mortgage-equity capitalization models are the most widely used income property valuation models."<sup>6</sup> Lusht reports that a mortgage-equity model appeared in 72 percent of the appraisals surveyed, with an equity yield version (such as the Ellwood model) appearing twice as often as the equity dividend version (i.e., the McLaughlin model) in which the overall rate is a weighted average of the equity dividend rate and the mortgage constant.

#### The McLaughlin Model

The McLaughlin model is perhaps better known as a simple mortgage-equity or band-of-investment model.<sup>7</sup> The McLaughlin model is expressed as follows:<sup>8</sup>

$$(2) \quad V = \frac{\text{NOI}}{(1 - M)\text{EDR} + Mf} \quad .$$

Without further statements of how the equity dividend rate (EDR), mortgage constant (f), and loan-to-value ratio (M) are determined, equation (2) is simply a definition. By substituting the definition of equity dividend rate into equation (2), it is a simple matter to show that (2) is an identity.<sup>9</sup> That is, the value estimate will be exactly the price paid if EDR, M, and f are exactly as occurs. However, by assuming that M is the maximum allowed by lenders (i.e., as



in the Ellwood model) and EDR is the minimum accepted (i.e., viewed as a hurdle rate) by equity investors, estimated value can differ from the selling price. In this paper, it is assumed that an operational McLaughlin model uses these constrained magnitudes for M and EDR.

#### The Gettel Model

The Gettel model sparked much of the recent interest in lender criteria. It did this by focusing on two lender underwriting criteria, the debt coverage ratio and the loan-to-value ratio, in addition to the mortgage constant. The Gettel model is expressed as follows:<sup>10</sup>

$$(3) \quad V = \frac{NOI}{(DCR)fM}$$

Without further statements of how the debt-coverage ratio (DCR), mortgage constant (f), and loan-to-value ratio (M) are determined, equation (3) is also simply a definition. By substituting the definition of debt coverage ratio into equation (3), it becomes clear that (3) is an identity.<sup>11</sup> The debt-coverage ratio in an operational Gettel model is assumed to be the minimum allowed by lenders. This is the second of the lender criteria dealt with in this paper. Assumptions regarding net operating income, the mortgage constant, and the loan-to-value ratio are the same as for the Ellwood and McLaughlin models.

#### The Lusht-Zerbst Model

The Lusht-Zerbst model maintains the emphasis on the debt coverage ratio begun by Gettel but does so in the context of a discounted cash flow model. A computational version of the Lusht-Zerbst model is expressed as follows:<sup>12</sup>

$$(4) \quad V = \frac{\text{NOI}}{(\text{DCR})f \left[ \frac{(1+y)^n - (1-\text{dep})}{(1+y)^n + f s_n^y (\text{DCR} - 1) - (1-P)} \right]}$$

Equation (4) is not simply a definition; the expectations and requirements of equity investors play an explicit role in this model, as they do in the Ellwood model.

The Lusht-Zerbst model differs from the Ellwood model in that it assumes that the loan amount is not explicitly a function of the loan to value ratio (M), as in Ellwood, but of the debt coverage ratio (DCR). The model utilizes the Gettel notions that NOI/DCR(f) equals the loan amount and DCR is the minimum allowed by the lender. The Lusht-Zerbst model is similar to the Ellwood model in its assumptions regarding f, y, NOI, n, and dep.

#### The Steele Model

The Steele model incorporates one lender criterion and the equity investor criterion. The Steele model may be expressed as follows:<sup>13</sup>

$$(5) \quad V = \frac{\text{NOI}}{\frac{\text{EDR}(\text{DCR})f}{\text{EDR} + (\text{DCR} - 1)f}}$$

Without further statements of how EDR, DCR and f are determined, equation (5) is simply a definition, similar to the McLaughlin and Gettel models. Here it is assumed that the EDR is the minimum acceptable to the equity investor, the DCR is the minimum allowable by the lender, and f is exogenously provided by the credit market.

#### The Fisher-Lusht Model

As a follow-up to the Lusht-Zerbst model, Fisher and Lusht present a method for selecting the minimum DCR that is interesting but enjoys

neither empirical nor theoretical support as of this writing. This method is incorporated in the Lusht-Zerbst model and treated here as a separate model, expressed as follows:<sup>14</sup>

$$(6) \quad V = \frac{\text{NOI}}{f \left[ \frac{y(1-M)}{Mf} + 1 \right] \left[ \frac{(1+y)^n - (1-\text{dep})}{(1+y)^n + y s_n^y \frac{(1-M)}{M} - (1-P)} \right]}$$

While M is the maximum allowed by lenders, the model generally implies a resulting loan-to-value ratio [equal to the third expression in the denominator of the right side of equation (6)] that is less than M.

Fisher and Lusht assume a conservative version of the project to be valued in which income growth is zero and the reversion proceeds equal the original equity (i.e., any build-up in equity through payments on the mortgage principal is offset by depreciation in the property value). With these assumptions, it can be shown that the equity yield rate (y) is equal to the equity dividend rate (EDR).<sup>15</sup> Fisher and Lusht use their conservative assumptions to derive a DCR [equal to the second expression in the denominator of the right side of equation (6)] which can then be used in the Lusht-Zerbst model, equation (4). This is equivalent to using equation (6).

#### The Cannaday-Colwell Model

One more model, in addition to the six presented above, is needed to complete the analysis of the lender and equity investor criteria under consideration. Like the Ellwood, Lusht-Zerbst, and Fisher-Lusht models, this model is based on discounted cash flows. This model will be called the Cannaday-Colwell model and is expressed as follows:<sup>16</sup>

$$(7) \quad V = \frac{\text{NOI}}{\text{EDR} \left[ \frac{(P - \text{dep})/s_n^y}{y - \text{EDR} + P/s_n^y} \right] + f \left[ \frac{y - \text{EDR} + \text{dep}/s_n^y}{y - \text{EDR} + P/s_n^y} \right]}$$

The Cannaday-Colwell model utilizes the equity dividend rate in much the same way as the Ellwood model utilizes the loan-to-value ratio and the Lusht-Zerbst model utilizes the debt coverage ratio. In the subsequent graphical analysis it is shown that the Cannaday-Colwell model is applicable if the equity investor's EDR requirement is the binding constraint (rather than the lender's constraints on M or DCR).

#### Definitional vs. DCF-based Models

The models presented above can be classified into two basic categories (see Table 2). The McLaughlin, Gettel, and Steele models are definitional in origin as previously discussed. The three definitional models cover all the possible combinations of the constraints (M, DCR, and EDR) taken two at a time. The other four models (i.e., Ellwood, Lusht-Zerbst, Fisher-Lusht, and Cannaday-Colwell) are computational versions of discounted cash flow (DCF) models. The implications of these differing origins will become clear in the graphical analysis which follows.

#### Graphical Analysis

The analytical device utilized is a two-dimensional graph of certain elements in the valuation process. A key lender criterion is on each axis of the graph; loan-to-value ratio (M) on the vertical axis and debt coverage ratio (DCR) on the horizontal axis. For each graph, a specific mortgage constant (f), net operating income (NOI), equity yield rate (y),

TABLE 2

## Summary Of Real Estate Valuation Models

Model	Value Equation	Constraints
<u>Definitional</u>		
McLaughlin	$V = \frac{\text{NOI}}{(1 - M)\text{EDR} + \text{MF}}$	M, EDR
Gettel	$V = \frac{\text{NOI}}{(\text{DCR})fM}$	M, DCR
Steele	$V = \frac{\text{NOI}}{\frac{\text{EDR}(\text{DCR})f}{\text{EDR} + (\text{DCR} - 1)f}}$	DCR, EDR
<u>Discounted Cash Flow</u>		
Ellwood	$V = \frac{\text{NOI}}{(1 - M)y + \text{MF} - \text{MP}/s_n^y + \text{dep}/s_n^y}$	M
Lusht-Zerbst	$V = \frac{\text{NOI}}{(\text{DCR})f \left[ \frac{(1 + y)^n - (1 - \text{dep})}{(1 + y)^n + fs_n^y(\text{DCR} - 1) - (1 - P)} \right]}$	DCR
Fisher-Lusht	$V = \frac{\text{NOI}}{f \left[ \frac{y(1 - M)}{\text{MF}} + 1 \right] \left[ \frac{(1 + y)^n - (1 - \text{dep})}{(1 + y)^n + y s_n^y \frac{M}{(1 - M)}} - (1 - P) \right]}$	$\text{DCR} = \frac{y(1 - M)}{\text{MF}} + 1$
Cannaday-Colwell	$V = \frac{\text{NOI}}{\text{EDR} \left[ \frac{(P - \text{dep})/s_n^y}{y - \text{EDR} + P/s_n^y} + f \right] + \frac{y - \text{EDR} + \text{dep}/s_n^y}{y - \text{EDR} + P/s_n^y}}$	EDR



holding period ( $n$ ), and depreciation in value ( $dep$ ) are assumed. Iso-value curves, a project line, and iso-equity dividend rate curves are defined. The feasible set consistent with lender and equity investor criteria is identified. Finally, the M-DCR combinations and the value estimates for each model are illustrated.

### Iso-Value Curves

An iso-value curve includes all the combinations of the loan-to-value ratio ( $M$ ) and the debt coverage ratio ( $DCR$ ) that produce the same value. The Gettel value equation,  $V = NOI/[DCR(f)M]$ , indicates that given  $NOI$  and  $f$ , value varies inversely with the product of  $M$  and  $DCR$ .<sup>17</sup> Therefore, by finding a curve in M-DCR space where the product of  $M$  and  $DCR$  is constant, one identifies the various combinations of  $M$  and  $DCR$  that produce equal value. A curve that is the locus of these combinations is commonly called a rectangular hyperbola. The term used in this paper is iso-value curve in order to stress its economic interpretation.

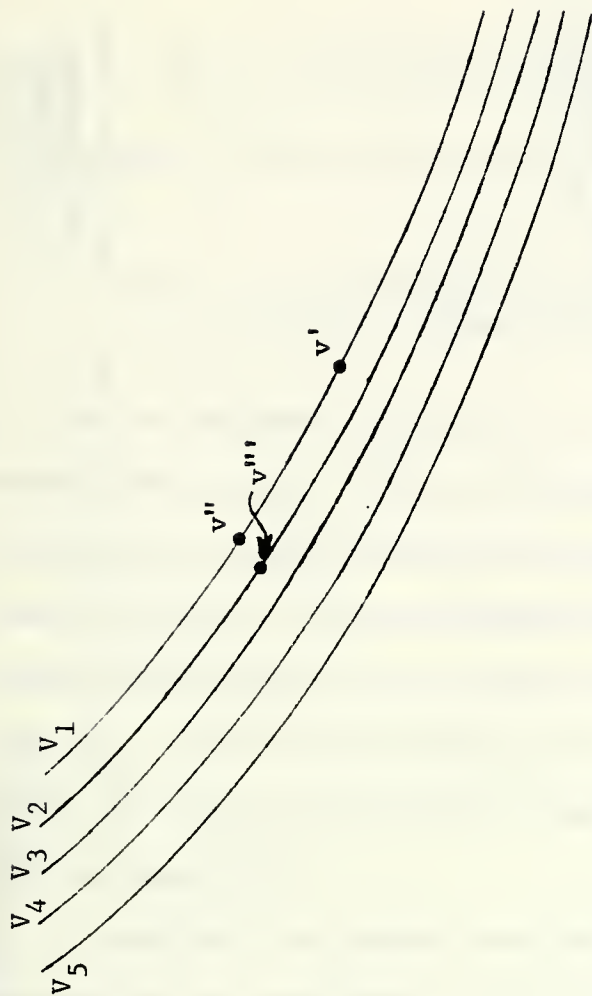
A lower iso-value curve indicates a higher value because of the inverse relationship between value and the product of  $M$  and  $DCR$ . That is, point  $v'$  on  $V_1$  in Figure 1 produces the same value as  $v''$  on  $V_1$  but a lower value than  $v'''$  or any other point on  $V_2$ . Note that  $v'''$  has lower  $DCR$  but higher  $M$  than  $v'$ . It is clear, however, that  $v'''$  indicates a higher value than  $v'$  because  $v'''$  has lower  $M$  and lower  $DCR$  and thus higher value than  $v''$ , and  $v'$  has the same value as  $v''$ .

### Project Line

A project line, such as shown in Figure 2, reveals the equity investor's view of combinations of  $DCR$  and  $M$ . It may be derived from the Ellwood model as follows:<sup>18</sup>



Loan-to-Value Ratio (M)

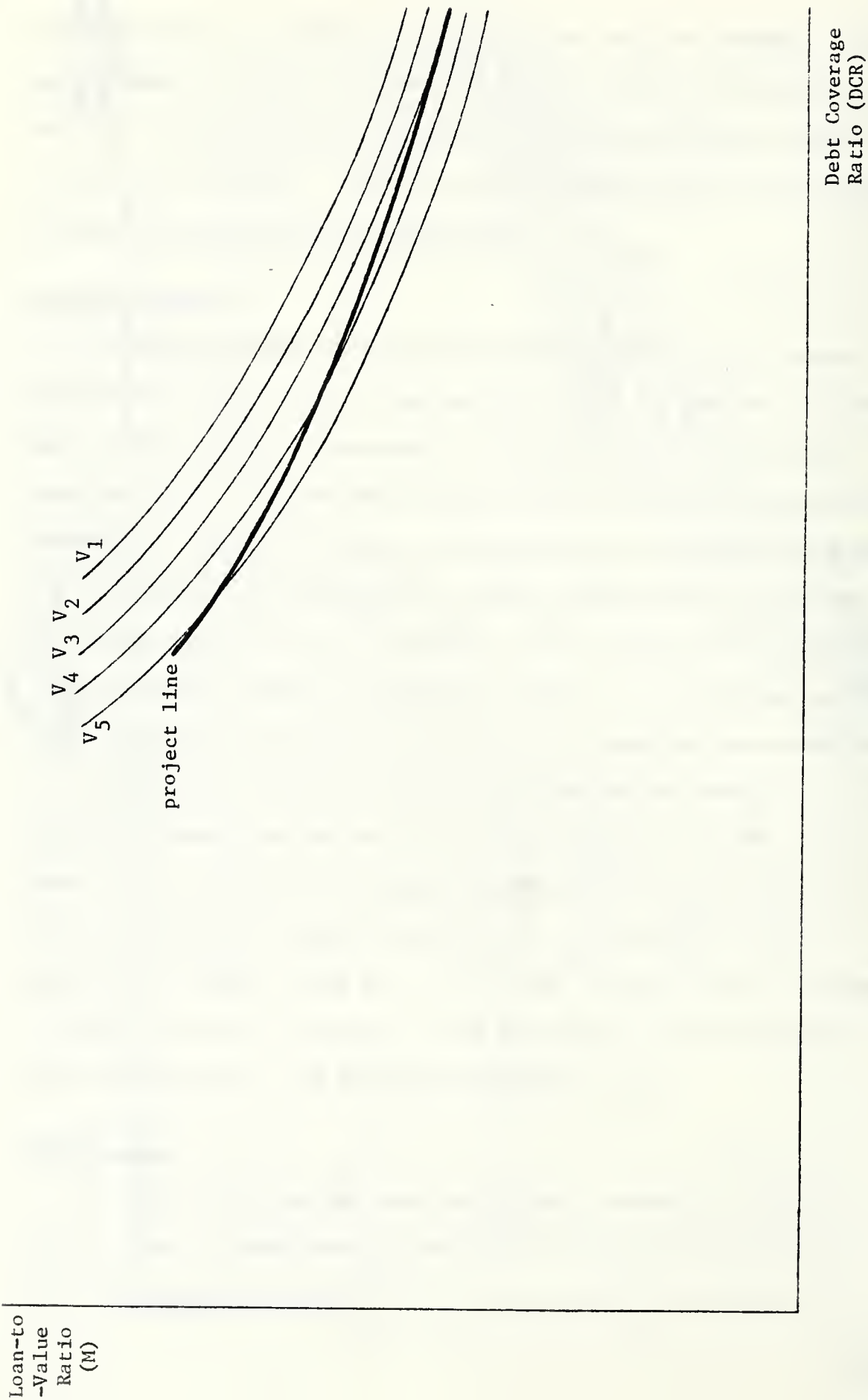


$$V_5 > V_4 > V_3 > V_2 > V_1$$

Debt Coverage Ratio (DCR)

Iso-Value Curves

Figure 1



Project Line

Figure 2

$$(8) \quad DCR = 1 + (1/f)[(y/M) - y - (P/s_n^y) + (dep/Ms_n^y)]$$

Alternatively, it may be derived from the Lusht-Zerbst model as follows:<sup>19</sup>

$$(9) \quad M = \frac{(1 + y)^n - (1 - dep)}{(1 + y)^n + f s_n^y(DCR - 1) - (1 - P)}$$

It is completely irrelevant for the position of the project line which method is used. However, the meaning of a point on the project line depends on whether one takes an Ellwoodian or Lusht-Zerbstian view of the world. From an Ellwoodian view, a point indicates the project's DCR given the M and other relevant magnitudes (i.e., f, y, dep, etc.). From a Lusht-Zerbstian view, a point on the project line yields the resulting M from having pre-selected the DCR, given other relevant magnitudes.

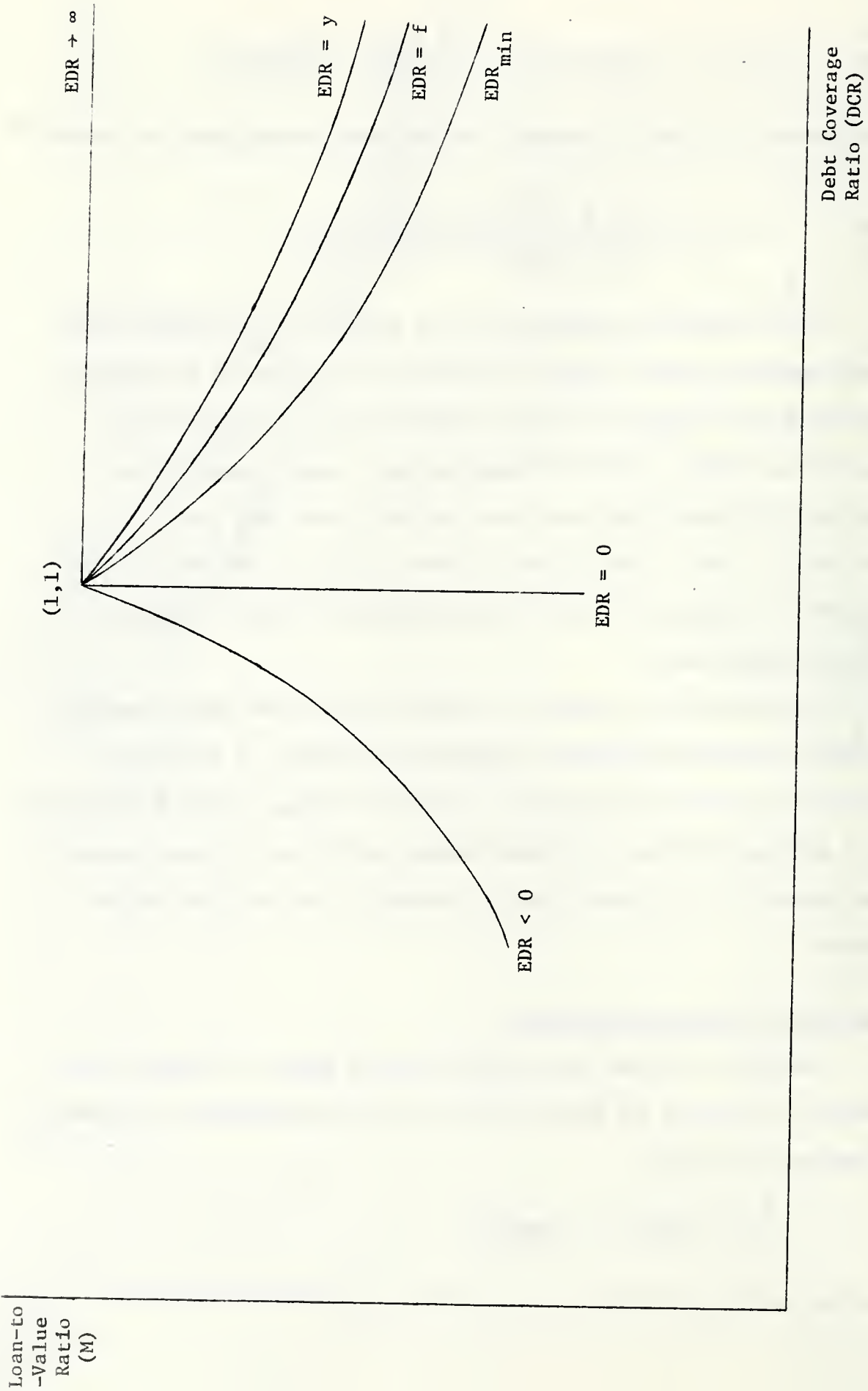
In addition, the project line reflects lender and equity investor expectations and requirements concerning key aspects of the project. These expectations and requirements include mortgage terms (as reflected by a mortgage constant), projected equity yield rate, holding period, and appreciation or depreciation in property value over the holding period.

#### Iso-Equity Dividend Rate Curves

Iso-equity dividend rate (EDR) curves, as shown in Figure 3, are derived by equating the overall rates in the McLaughlin and Gettel value equations as follows:

$$(1 - M)EDR + Mf = DCR(f)M$$

Solving for DCR yields:



Iso-EDR Curves

Figure 3

$$(10) \quad DCR = EDR(1-M)/Mf + 1.$$

Equation (10) is then used to develop each iso-EDR curve by assuming a particular EDR, allowing M to vary, and solving for DCR (given a constant f). Resulting pairs of M and DCR can then be used to plot the related iso-EDR curve.

All iso-EDR curves approach the point (1,1) as shown in Figure 3. The slope of an iso-EDR curve has the opposite sign of EDR (i.e., positive when EDR is negative and vice-versa). When  $EDR = f$ , the iso-EDR curve is an iso-value curve also.

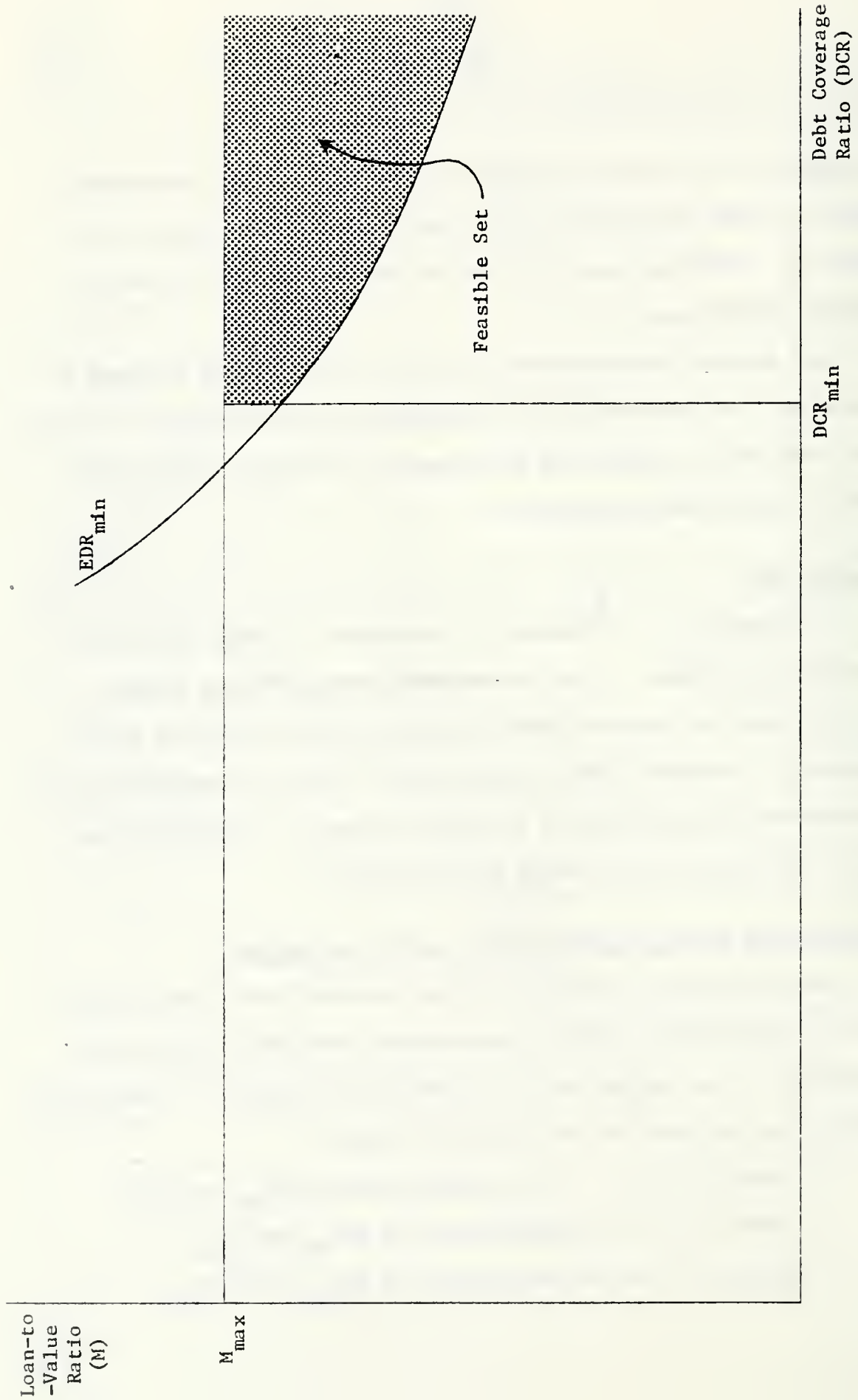
#### Feasible Set

The feasible set represents an integration of lender and equity investor constraints. A view of mortgage lending in which lenders restrict loans to projects having a specific DCR or higher and simultaneously to projects having a specific M or lower is incorporated with the minimum EDR requirement of the equity investor. The feasible set may be illustrated as the shaded area in Figure 4.

#### Loan-to-Value Ratio and Debt Coverage Ratio Combinations

The loan-to-value ratio (M) and debt coverage ratio (DCR) combination for each model is shown by superimposing the project line on the boundaries of the feasible set, as illustrated in Figure 5. These M-DCR combinations are found for each model as follows:

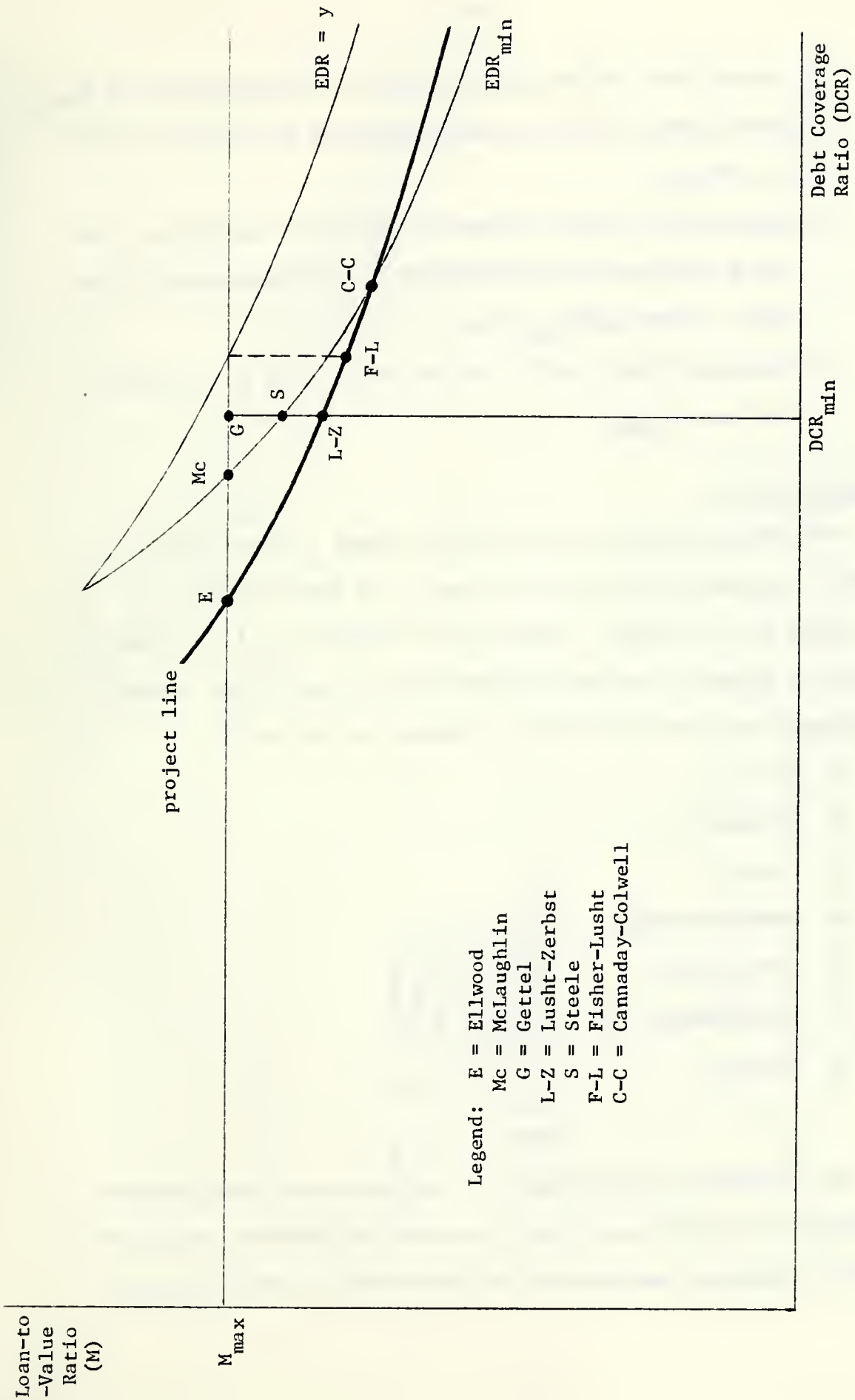
- 1) McLaughlin (Mc) - at the intersection of  $EDR_{min}$  and  $M_{max}$ ;
- 2) Gettel (G) - at the intersection of  $DCR_{min}$  and  $M_{max}$ ;
- 3) Steele (S) - at the intersection of  $DCR_{min}$  and  $EDR_{min}$ ;



Feasible Set

Figure 4





M-DCR Combinations

Figure 5

- 4) Ellwood (E) - at the intersection of the project line and  $M_{\max}$ ;
- 5) Lusht-Zerbst (L-Z) - at the intersection of the project line and  $DCR_{\min}$ ;
- 6) Fisher-Lusht (F-L) - at the intersection of the project line and a "conservative" DCR (defined by the intersection of the  $EDR = y$  curve and  $M_{\max}$ ); and
- 7) Cannaday-Colwell (C-C) - at the intersection of the project line and  $EDR_{\min}$ .

#### Value Estimates

Differences among the value estimates based on each model are shown by superimposing iso-value curves on the M-DCR combinations identified for each model. These value estimates are illustrated in Figure 6. By model, the value estimates for the particular example illustrated here rank from lowest to highest as follows:<sup>20</sup>

- 1) Gettel;
- 2) McLaughlin;
- 3) Steele;
- 4) Cannaday-Colwell;
- 5) Fisher-Lusht;
- 6) Lusht-Zerbst; and
- 7) Ellwood.

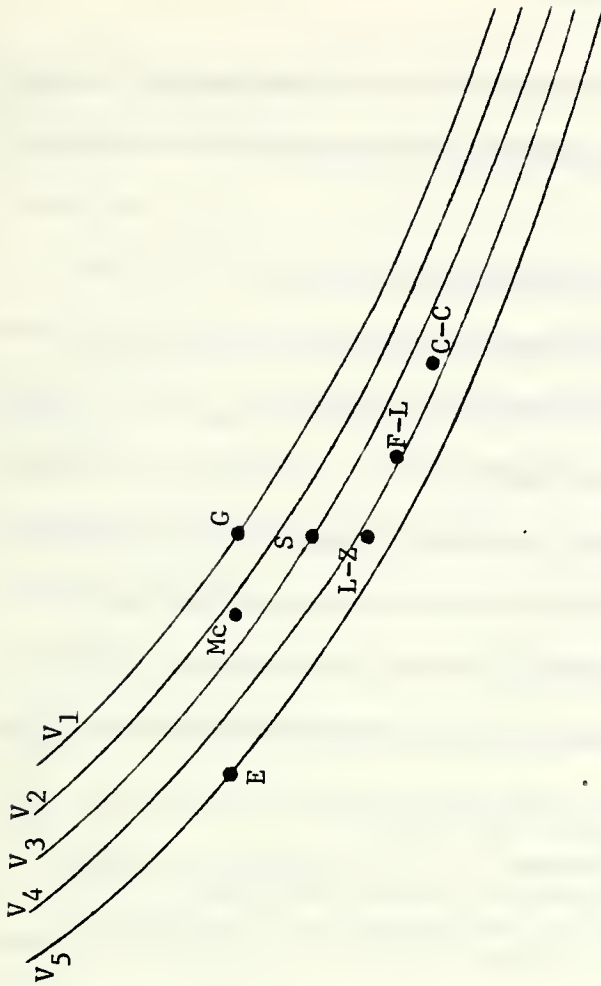
#### Model Choice

The criterion for model choice is that the model which generates the highest value estimate without violating any lender or equity investor constraints, expectations, or requirements is the correct one.

Loan-to-Value Ratio (M)

$$V_5 > V_4 > V_3 > V_2 > V_1$$

Legend: E = Ellwood  
Mc = McLaughlin  
G = Gettel  
L-Z = Lusht-Zerbst  
S = Steele  
F-L = Fisher-Lusht  
C-C = Cannaday-Colwell



Debt Coverage Ratio (DCR)

Value Estimates

Figure 6

From the previous graphical analysis it can be seen that several models violate lender or equity investor constraints for the specific example illustrated, as follows:

- 1) McLaughlin - violates DCR constraint;
- 2) Fisher-Lusht - violates EDR constraint;
- 3) Lusht-Zerbst - violates EDR constraint; and
- 4) Ellwood - violates DCR constraint.

The models which violate lender or equity investor expectations or requirements (i.e., those which generate value estimates that do not lie on the project line) are McLaughlin, Gettel, and Steele. By a process of elimination, it is shown for this example that the Cannaday-Colwell model is the only one that does not violate any constraint and meets all expectations and requirements; i.e., it generates a value estimate which falls on a boundary of the feasible set and lies on the project line.

The Cannaday-Colwell model is not always the correct choice. An example can be developed for which the Cannaday-Colwell model violates one of the lender constraints. In such an example, either the Ellwood or Lusht-Zerbst models would turn out to be correct. The definitional models (McLaughlin, Gettel, and Steele) produce value estimates which fall on the project line only by rare coincidence. However, the definitional models may produce the correct value without falling on the project line, again by coincidence. The Fisher-Lusht model yields a value estimate that will always fall on the project line. However, it will generally violate one of the constraints or produce a lower value estimate than another model which falls on the

project line and does not violate any constraints. The only other possibility is that the Fisher-Lusht and Lusht-Zerbst value estimates coincide.

Either the Colwell-Cannaday, the Lusht-Zerbst, or the Ellwood model will be correct depending upon whether the binding constraint is  $EDR_{\min}$ ,  $DCR_{\min}$ , or  $M_{\max}$ , respectively. This can be determined by graphical analysis as previously illustrated. Alternatively, the one of these three models that yields the lowest value estimate will be the one that does not violate any constraints. Therefore, the correct model can be selected by estimating value using each of the three value equations and determining which yields the lowest value estimate.

#### Summary and Conclusions

Several graphical devices are developed that facilitate the integration of lender and equity investor criteria with discounted cash flow models. These devices include iso-value curves, project lines, and constraints on the loan to value ratio, the debt coverage ratio, and the equity dividend rate.

It is found that the definitional models (i.e., McLaughlin, Gettel, and Steele), in the context developed here, are correct only by mere coincidence. Simultaneous consideration of two of the three key criteria, as in the definitional models, completely eliminates the possibility of consideration of anything else; i.e., the models become tautological. The Fisher-Lusht model is dismissed because, in general, it violates one of the constraints or produces a lower value estimate than another model which falls on the project line and does not violate any constraints.

A new valuation model based on discounted cash flow is developed to fill a gap in the literature that becomes evident in the graphical analysis. It is shown that this new Cannaday-Colwell model and the Ellwood and Lusht-Zerbst discounted cash flow based models each tell one-third of the story (i.e., each considers a different constraint). One of these three discounted cash flow models will be correct depending upon whether the binding constraint is  $M_{\max}$ ,  $DCR_{\min}$ , or  $EDR_{\min}$ . The relevant model is the one that yields the lowest value estimate of the three.

The graphical analysis is illustrated for one possible combination of constraints and project line. We leave it to the reader to experiment with alternative constraints and project lines. For example, what would the feasible set look like if  $DCR_{\min}$  is less than unity while the  $EDR_{\min}$  is slightly negative. The graphs are useful in noting the value effects of changing the constraints or changing the environment of expectations given fixed constraints. Allowing the depreciation to vary is a simple way to cause the project line to shift and cause the binding constraint to change. As the binding constraint changes, the graphical analysis indicates how model choice changes.

The discounted cash flow models considered in this paper may be criticized on the basis of their simple assumptions about paths of income and debt service. However, it should be recognized that these models readily admit to much more complication in these particulars without changing their fundamental character substantially. The simplicity of the computational forms presented here already disguises an elaborate discounted cash flow parentage. It is well-known how net



operating income can be made to change linearly or exponentially or how to include selling expenses, loan discount points and prepayment penalties.<sup>21</sup> It is not much more complex to add periodic rent renegotiations with intervening step-ups or step-downs.<sup>22</sup> Similarly, more complex mortgage contracts such as graduated payment mortgages could be incorporated.<sup>23</sup> None of these complications fundamentally affect the conclusions of this paper. That is, regardless of the particular cash flow path, there will be a project line that cuts the feasible set thereby revealing which constraint is binding, which valuation model is correct, and the value of the project. Where this approach can be faulted is in the assumption that there are constraints that bind. An alternative view would be that there is an optimal capital structure and that when equity investors feel bound by institutional lenders' constraints on loan-to-value ratio or debt coverage ratio, they simply go elsewhere.

FOOTNOTES

<sup>1</sup>See Institutional Investor, Inc., Institutional Investor, NY: Gilbert E. Kaplan, Publisher, June 1981, pp. 95, 104, and 118 for an indication of the "hurdle" rates established by several large institutional investors.

<sup>2</sup>From the definition of equity dividend rate given in Table 1, it can be seen that when net operating income (NOI) and debt service (MVf) are constant, the equity dividend rate is simply the reciprocal of the payback period.

<sup>3</sup>L. W. Ellwood, Ellwood Tables for Real Estate Appraising and Financing, Ridgewood, N.J.: L. W. Ellwood, 1959; Frank J. McLaughlin, "Proper Capitalization Rates," The Appraisal Journal, Oct. 1959, pp. 543-547; Ronald E. Gettel, "Good Grief, Another Method of Selecting Capitalization Rates?!", The Appraisal Journal, Jan. 1978, pp. 90-100; Kenneth M. Lusht and Robert H. Zerbst, "Valuing Income Property in an Inflationary Environment," The Real Estate Appraiser and Analyst, Jul.-Aug. 1980, pp. 11-17; Robert A. Steele, "DCR/R Capitalization Rate Tables for Today's Financing," The Appraisal Journal, Jan. 1981, pp. 15-22; and, Jeffrey D. Fisher and Kenneth M. Lusht, "Mortgage Equity Analysis with a Debt Coverage Constraint," The Real Estate Appraiser and Analyst, Fall 1981, pp. 5-12.

<sup>4</sup>L. W. Ellwood, Ellwood Tables for Real Estate Appraising and Financing, 4th ed., Cambridge, Mass.: Ballinger Publishing Co., 1977, p. 11. Note that  $d = NOI$ . The exact formulation of equation (1) and all subsequent equations is consistent with that used in Roger E. Cannaday and Peter F. Colwell, "A Unified Field Theory of the Income Approach to Appraisal," The Real Estate Appraiser and Analyst, First Quarter 1981, pp. 5-9 (Part 1), Summer 1981, pp. 29-43 (Part 2), and Fall 1981, pp. 25-37 (Part 3).

<sup>5</sup>For a discussion of the concept of marginal investor see: (1) Peter F. Colwell and James R. Webb, "Typical Investors, Marginal Investors and Market Value," The Real Estate Appraiser and Analyst, Nov.-Dec. 1980, pp. 13-15 and; (2) Paul F. Wendt, Real Estate Appraisal: Review and Outlook, Athens, Ga.: University of Georgia Press, 1974, p. 7.

<sup>6</sup>Kenneth M. Lusht, "The Behavior of Appraisers in Valuing Income Property: A Status Report," The Real Estate Appraiser and Analyst, Jul.-Aug. 1979, p. 50.

<sup>7</sup>It is called simple mortgage-equity in William N. Kinnard, Jr., Income Property Valuation, Lexington, Mass.: Heath Lexington Books, 1971, p. 265. It is called a band of investment technique in Textbook Revision Committee, The Appraisal of Real Estate, 7th ed., Chi: AIREA, 1978, p. 395.

<sup>8</sup>McLaughlin, "Proper Capitalization Rates," p. 545. See Cannaday and Colwell, "A Unified Field Theory...", Part 3, p. 26 for the formulation of equation (2) presented here. Also, see Lusht, "The Behavior of Appraisers in Valuing Income Property...", p. 52 for a brief discussion of McLaughlin's contribution.

<sup>9</sup>If the definition of equity dividend rate [ $EDR = (NOI - MVf)/(1 - M)V$ ] is substituted for EDR in equation (2):

$$V = \frac{NOI}{(1 - M)[(NOI - MVf)/(1 - M)V] + Mf} = V$$

<sup>10</sup>Gettel, "Good Grief, Another Method ...," p. 100. See Cannaday and Colwell, "A Unified Field Theory...", Part 3, p. 27 for the formulation of equation (3) presented here.

<sup>11</sup>If the definition of debt coverage ratio ( $DCR = NOI/MVf$ ) is substituted for DCR in equation (3):

$$V = \frac{NOI}{(NOI/MVf)(f)M} = V$$

<sup>12</sup>Lusht and Zerbst, "Valuing Income Property in an Inflationary Environment," p. 15. See Cannaday and Colwell, "A Unified Field Theory...", Part 3, p. 28 for the formulation of equation (4) presented here. Equation (4) is equivalent to the discounted cash flow version presented by Lusht and Zerbst.

<sup>13</sup>Steele, "DCR/R Capitalization Rate Tables for Today's Financing," p. 16. The formulation of equation (5) presented here is equivalent to NOI divided by the overall rate (R) derived by Steele.

<sup>14</sup>Fisher and Lusht, "Mortgage Equity Analysis with a Debt Coverage Constraint," p. 7 (especially footnote 5). Fisher and Lusht do not explicitly present a value equation for their method. The formulation of equation (6) is implicit in their method and is developed here in a manner consistent with the value equations for the other models presented. To derive equation (6), substitute y for EDR in equation (10) to yield the conservative DCR. Substitute the resulting expression for the conservative DCR in equation (4) and simplify.

<sup>15</sup>See Kinnard, Income Property Valuation, p. 265.

<sup>16</sup>Equation (7) is derived by first equating the right sides of equations (8) and (10) and solving for M. Then the expression for M is substituted in equation (2) and simplified.

<sup>17</sup>Remember that the Gettel value equation is more than just a definition when you give behavioral dimensions to selecting M and DCR. The Gettel value equation, itself, is always true and is implicit in the iso-value curves.

<sup>18</sup>Equation (8) is derived by equating the right sides of equations (1) and (3) and solving for DCR.

<sup>19</sup>Equation (9) may be derived by equating the right sides of equations (3) and (4) and solving for M.

<sup>20</sup>The example used to illustrate the graphical analysis assumes the following: NOI = \$111,450; dep = -0.475; y = 0.20; n = 7 yrs.; i = 0.15; and m = 25 yrs. (annual payments). The value estimates derived are as follows: Gettel = \$720,400; McLaughlin = \$775,300; Steele = \$799,300; Cannaday-Colwell = \$836,400; Fisher-Lusht = \$847,400; Lusht-Zerbst = \$857,000; and Ellwood = \$900,500.

<sup>21</sup>See Cannaday and Colwell, "A Unified Field Theory...", Part 2, pp. 36-43 for a discussion of how net operating income can be made to change linearly or exponentially. See Cannaday and Colwell, "A Unified Field Theory...", Part 1, p. 8 for a discussion of how to handle selling expenses, loan discount points and prepayment penalties.

<sup>22</sup>See C. G. Blackadar, Dynamic Capitalization: Appraising with Real Rates of Interest, N.Y.: Metropolitan Life Insurance Co., 1981, p. 8.

<sup>23</sup>Also, it would be possible to develop the graphical analysis with a third lender criteria such as term of the mortgage (m) as the third dimension for the graphs. The mortgage interest rate (i) would then be assumed to be exogenously determined in the credit market [instead of assuming the mortgage constant (f) is exogenously determined].



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